A drone routing problem.

The problem is inspired by that presented in a paper by Kyriakakis, Stamadianos, Marinaki, Marinakis (2022), who proposed heuristics.

Data.

- a set N of customers in known positions;
- the weight w_i of the package to be delivered to each customer $i \in N$;
- a set of identical electric vehicles characterized by
  - a capacity Q^V;
  - a battery capacity B^V;
  - an energy consumption coefficient π^V;
- a set P of launch/retrieval locations;
- a set of identical drones with
  - a weight T;
  - a capacity Q^D (maximum allowed payload);
  - a battery capacity B^D;
  - an energy consumption coefficient π^D;
- the distances d between all relevant locations.

Constraints.

All customers must be served: each of them requires one of the packages.

Each drone must return to the same location from which it has been launched. Each EV remains in the same location until all its drones have come back to it.

Drones may travel multiple routes from the same launch/retrieval location.

The energy consumption of vehicles along any arc (i, j) of the graph is $\pi^V d_{ij}$ (the load is negligible compared to the weight of the vehicle), while the energy consumption of each drone along an arc (i, j) is computed as the product between the distance travelled d_{ij} and the total weight transported along it, including the drone weight T and its payload f_{ij} (which depends on the arc), i.e. $d_{ij}(T + f_{ij})$.

All operations different from traveling are assumed not to consume energy.

Objective.

Minimize the total energy consumption.

An extended formulation.

The proposal is to develop a column generation algorithm to solve the linear relaxation of an extended formulation. This can be used to set up a math-heuristic using math programming solvers as well as for a branch-and-price algorithm.

Master problem.

$$
\begin{aligned}
\text{minimize } z = \sum_{k \in K} c'_k \theta_k + \sum_{p \in p, p \neq 0} \sum_{h \in H_p} c''_h \phi_h & & (1)\\
\text{s.t.} \quad \sum_{p \in P, p \neq 0} \sum_{h \in H_p} a_{ih} \phi_h \geq 1 & \quad \forall i \in N & (2)\\
\sum_{h \in H_p} \omega_h \phi_h = \sum_{k \in K} v_{pk} \theta_k & \quad \forall p \in P, p \neq 0 & (3)\\
\theta_k \text{ binary} & \quad \forall k \in K & (4)\\
\phi_h \text{ binary} & \quad \forall p \in P, p \neq 0, \forall h \in H_p & (5)
\end{aligned}
$$

K is the (exponentially large) set of feasible vehicle routes. H_p is the (exponentially large) set of feasible drone routes based at location p.

Each variable θ_k indicates the selection of a vehicle route. Each variable ϕ_h indicates the selection of a drone route. Binary restrictions on these variables are to be relaxed and column generation is used to solve the resulting LP model.

Coefficients have the following meaning:

- c'_k: energy consumption of vehicle route k;
- c''_h: energy consumption of drone route h;
- a_{ih}: equal to 1 if drone route h visits customer i, 0 otherwise;
- ω_h: total load delivered by drone route h;
- v_{pk}: total load carried from vehicle route k to location p.

Pricing.

Columns are priced out by two pricing algorithms, one for columns in K and the other for columns in H_p for each p. For moderate size instances a general-purpose MILP solver may suffice. For large scale instances we can develop exact and heuristic specialized algorithms.